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FRECHET DIFFERENTIABILITY OF A FIELD OPERATOR FOR SCATTERING FROM AN OPEN SCREEN

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ABSTRACT

The inverse problem, which we consider is to determine the shape of two-dimensional open screen from the knowledge of the field on a curve for the electromagnetic plane waves scattering.

Prof. R. Kress in his papers has proposed to reconstruct the scatterer's - shape from the knowledge of the far field pattern [1-2]. We extend this approach to the inverse problem of determining the shape of a two-dimensional open scatterer from the knowledge of the scattered field on a curve. In particular, we investigate the Frechet differentiability of a field operator for scattering from an open screen with the boundary as prerequisite for the theoretical foundation of the gradient methods or Newton type methods for the approximate solution of this nonlinear and improperly posed problem.

The aim of this paper is to provide a proof for Frechet differentiability with respect to the boundary of an operator, which maps the boundary of an open screen onto the scattered field and to obtain expression of this derivatives.

STATEMENT OF THE PROBLEM

We consider the scattering of time-harmonic electromagnetic plane waves by a thin infinitely long cylindrical screen with a cross which is section described by an arc $l \in R^2$ class $C^3[a,b]$, i.e. $l = \{(x, f(x)) : x \in [a,b]\}$ where $f(x)$ is an injective and three continuously differentiable function. The inverse problem consists in determination the shape of the screen from the knowledge of the field on curve S. Mathematically this problem can be interpreted as the solution with respect to l of a system operator equation

$$\begin{cases} \int\limits_l j(s')H_0^{(1)}(kr)ds' = g(z), & z \in S, \\ \int\limits_l j(s')H_0^{(1)}(kr)ds' = u(z), & z \in l. \end{cases} \tag{1}$$

Here, $z = \{x, y\}$ - any point of R^2, s' - arc abscissa of a point $z' = \{x', y'\}$ of the contour l, $H_0^{(1)}(x)$ - Hankel function of zero order and of the first kind, $g(z)$ - the given function on curve S and $\overline{u}(s)$ - a value of the incident field at the points of curve l.

Let F the operator that maps a description $f(x)$ of some admissible scatterer onto the corresponding scattered field $g(z), z \in S$. In terms of this operator the inverse problem consists in solution of the nonlinear and ill-posed equation for the function f,

$$Ff = g \tag{2}$$

where $g(z)$ is a (measured) scattered field.
In order to use Newton's method for the approximate solution of (2) it is necessary to establish the differentiability of operator F with respect to l.

TECHNIQUE OF SOLUTION

To obtain characterizations of the derivatives we use a method, which is based on the classical theory of boundary integral operators [3]. From (1) we obtain

$$F = SK^{-1}R\overline{u}.$$

Here

$$K : C^{(0,\alpha)}(l) \Rightarrow C^{(1,\alpha)}(l),\ K(\varphi;l) \equiv \int_l \varphi(s')\Phi(z,z')ds',\ z \in l;$$

$$S : C^{(0,\alpha)}(l) \Rightarrow C^2(G),\quad S(\varphi;l) \equiv \int_l \varphi(s')\Phi(z,z')ds',\ z \in G;$$

$$R : C^2(R^2) \Rightarrow C^2(l),\ R(u;l) \equiv u\,|_l,$$

with $\Phi(z,z') = \dfrac{H_0^{(1)}(kr)}{\sqrt{|s'-z_1||s'-z_{-1}|}}$, where z_1 and z_{-1} are extremities of the arc l, G is any domain of R^2 for which $\overline{G} \cap l = \varnothing$.
The first step of the proof of a differentiability of the operator F consists in establishing that the mapping $f \Rightarrow K$ is Frechet differentiable from $C^3[a,b]$ into the space of bounded linear operators $L(C^{0,\alpha}[a,b], C^{1,\alpha}[a,b])$ and that the derivative is given by $h \Rightarrow K(\cdot; f,h)$, where $K(\cdot; f,h)$ denotes the integral operator

$$K'(\varphi; f,h)(x) = \int_a^b \frac{\varphi(\tau)}{\sqrt{|(\tau-a)(\tau-b)|}}\left[k_1(h;x,\tau) + k_2(h;x,\tau)\right]d\tau$$

with

$$k_1(h;x,\tau) = -kH_1^{(1)}(kr_f)J_f(\tau)\frac{f(x)-f(\tau)}{r_f}[h(x)-h(\tau)],$$

$$k_2(h;x,\tau) = H_0^{(1)}(kr_f)\frac{f'(\tau)}{J_f(\tau)}h'(\tau)$$

In the second step of the proof it is shown that mapping $f \Rightarrow S$ is Frechet differentiable from $C^3[a,b]$ into the space of bounded linear operators $L(C^{0,\alpha}[a,b], C^2[G])$ and that the derivative is given by $h \Rightarrow S'(\cdot; f,h)$, where $S(\cdot; f,h)$ denotes the integral operator

$$S'(\varphi; f,h)(x) = \int_a^b \frac{\varphi(\tau)}{\sqrt{|(\tau-a)(\tau-b)|}}\left[s_1(h;x,\tau) + s_2(h;x,\tau)\right]d\tau,$$

with

$$s_1(h;x,\tau)=-kH_1^{(1)}(kr)J_f(\tau)\frac{y-f(\tau)}{r}h(\tau),\ s_2(h;x,\tau)=H_1^{(0)}(kr)\frac{f'(\tau)}{J_f(\tau)}h'(\tau).$$

It is easily to convinced, that $f \Rightarrow R(\cdot;f)$ is Frechet differentiable one with the derivative $R'(u;f,h)=u'_y(x,f(x))h(x)$.

In the third step using the chain rule we finally obtain the Frechet differentiability of F. The derivative is given by

$$F'(f)h=S'(\phi_1;f,h)+S\left(K^{-1}(K'(\phi_1;f,h);f);f\right)+S(\phi_2;f)$$

where $\phi_1=K^{-1}(R(\overline{u};f);f)$, $\phi_2=K^{-1}(R'(\overline{u};f,h);f)$ For the actual numerical computation of the Frechet derivative of the operator F at first we determine $\phi_1(x)$ by solving the integral equation

$$\int_a^b \frac{\phi_1(\tau)}{\sqrt{|(\tau-a)(\tau-b)|}}H_0^{(1)}(kr_f)J_f(\tau)d\tau=R(\overline{u},f).$$

Further, we have to solve the integral equation

$$\int_a^b \frac{\phi_2(\tau)}{\sqrt{|(\tau-a)(\tau-b)|}}H_0^{(1)}(kr_f)J_f(\tau)d\tau=K'(\phi_1;f,h)+R'(u;f,h)$$

and at last we compute a sum

$$F'(f)h=S'(\phi_1;f,h)+S(\phi_2;f).$$

Hence, for numerical computation of the Frechet derivative it is necessary to solve two singular integral equations of the same type. The numerical methods for solving of such equations are known [4].

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